A PAIRWISE COMPARISON-BASED MODEL OF UNCERTAINTY IN MULTI-PERSON MULTI-CRITERIA DECISION PROBLEMS

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1. SUMMARY

This paper describes a simple model for the local uncertainty in a multi-person multi-criteria decision problem (MPMCDP). The model is motivated by the authors' experiences in the first stage of corporate innovation processes, which are characterized by a very large number of ideas, for which little or no clarifying information is available. Both the uncertainty model and the ranking algorithm are based on pairwise comparisons of alternatives in order to minimise the costs of processing ideas and to improve the reliability of the results. The model allows uncertainty to be detected cheaply and suggests an efficient method for its reduction.

2. INTRODUCTION

Innovation is a key factor in establishing and maintaining competitiveness. Many companies use innovation processes as a structured approach to obtaining new products and services. This process begins with the generation of a pool of ideas, may include activities such as evaluation, research, development and prototyping, and culminates in the launch of a new product. The process is often formulated using a Stage-Gate paradigm (Cooper, Edgett and Kleinschmidt 2001) (Belliveau, Griffin and Somermeyer 2002). In the first stage of this process, ideas for new or improved products are generated, and at the subsequent first gate, these ideas are evaluated, and those with the highest potential are selected to be investigated further.

Typical characteristics of the first gate of an innovation process are that a very large number of ideas can be involved, several decision-makers participate in the selection, and there are many selection criteria to be considered. We thus have to solve a large Multi-Person, Multi-Criteria Decision Problem (MPMCDP).

A major constraint of the early stages of an innovation process is that little or no background information is available on the ideas; the decision-makers must therefore make their judgements based on superficial descriptions of the ideas, which may only consist of a single sentence. Under such circumstances, decision-makers may consider different arguments and thus disagree in their judgements. This leads to uncertainty in the overall selection result. Uncertainty is stated to be a major difficulty in the innovation process (Leifer *et al* 2000). Some approaches to managing uncertainty in innovation processes are described in (Luoma, Paasi, Strong and Zhou, 2009). However, these approaches are only applicable in the later stages of the innovation process.

Selection can be subject to two types of error. With a rejection error, the selection procedure rejects an idea which – if it had been pursued – would have been successful. This results in lost opportunities. With an acceptance error, the selection procedure erroneously identifies an idea as (one of) the best in the pool. This leads to economic losses, since resources are invested in these ideas, which, however, ultimately prove to be unsuccessful in the market.

Clearly, additional information on the ideas such as technical feasibility, necessary investments and market attractiveness would aid the decision-makers in making better judgements. However, obtaining this information can be very expensive, since it requires experts and may involve substantial projects such as engineering studies or market research. It would therefore be prohibitively expensive and very inefficient to develop all the ideas in the first stage before evaluating them.

We are therefore faced with a dilemma: we need to identify the best ideas in the pool, but we cannot afford to generate the information that is needed to do so.

In the second and third authors' innovation consulting practice, this situation occurs frequently; because of the lack of additional information, raw ideas are debated controversially and receive widely varying evaluations. This observation formed the motivation to develop the solution described in this paper.

Based on a ranking algorithm which uses pairwise comparisons of alternatives, we have created a model which localises the uncertainty in the judgements. This allows us to restrict expensive further development of alternatives to those that are affected by the comparisons with the highest degree of uncertainty. The resulting procedure yields an overall decision that is more reliable than if no additional information is used, but at the same time can produce substantial savings compared to the brute-force approach in which all alternatives are developed before evaluation. The model is appropriate for use as a computer-based tool.

3. SCIENTIFIC PROBLEM

We are given an MPMCDP which corresponds to the first gate of an innovation process. This MPMCDP may typically involve from 5 to 10 decision-makers, from 3 to 15 decision criteria and up to several hundred alternatives. In the case of a product innovation application, the decision-makers represent various roles within a corporation such as management, marketing and engineering, the criteria include factors such as market attractiveness, technical feasibility and strategic fit, and the alternatives are raw ideas for new products that have been generated using, for example, market trends, creativity techniques or customer input.

The objective is to select a small number (typically 1 to 5) of alternatives that rank highest with respect to the set of criteria and include the opinions of all the decision-makers.

In the context of a given solution method for this MPMCDP, we wish to create a model of the uncertainty that is present in this selection. This model should localise the uncertainty and thus provide a clue as to which alternatives need to be developed further, with the expectation that a second evaluation of the developed alternatives will yield a selection result with a higher degree of certainty. Furthermore, the model should be simple to understand and to implement in a computer-based innovation management tool.

4. THE UNCERTAINTY MODEL

Our starting point for the model is the MPMCDP algorithm of Chelvier et al (Chelvier, Krull and Horton 2009) (Chelvier, Horton, Krull and Rauch-Gebbensleben 2009). This algorithm is based on pairwise comparisons of alternatives with respect to individual criteria. The results of these comparisons are then used to construct a Markov chain, in which each variable represents one of the alternatives and the arc weights represent the aggregated judgements. This Markov chain is then solved numerically using a Gauss-Seidel or Jacobi method (Stewart, 1994) to obtain a probability vector whose values are used to compute a ranking of the alternatives. The algorithm is similar in derivation and structure to the well-known PageRank algorithm (Page, Brin, Motwani and Winograd, 1999) which is used by the Internet search engine Google to compute rankings for web pages. The algorithm allows different coefficients to be assigned to represent varying degrees of importance both of the individual criteria and also of the judges.

Decision methods based on pairwise comparisons have a number of advantages over the more common scoring methods, which require decision-makers to assign numerical values to alternatives. These advantages are discussed, for example, in (Saaty and Sodenkamp 2008) and (Saaty 2008). Cooper *et al* also suggests that pairwise comparisons may be more appropriate than scoring when little information is available (Cooper, Edgett and Kleinschmidt 2001).

Our model is based on the assumption that decision-makers will give contradictory judgements when the information available for each of the alternatives under comparison is insufficient, since each decision-maker will base their judgement on the information available to themselves or on the arguments which occur to them spontaneously. On the other hand, if sufficient information is available, the decision-makers' decision will be unanimous, since we assume that decision-makers judge rationally and that a sufficient amount of information on the two alternatives under comparison will yield an unambiguous preference. (Our model ignores differences in personal taste, which will yield contradictory decisions regardless of the amount of information provided for each alternative.)

We denote the decision-makers by D_i, i=1..I, the alternatives by A_j, j=1..J and the evaluation criteria by C_k, k=1..K.

We now consider a pairwise comparison of the alternatives A_{j1} and A_{j2} with respect to criterion C_k. We collect a total of I judgements from the decision makers, of which I_{12} prefer A_{j1} over A_{j2} and I_{21} prefer A_{j2} over A_{j1}. Since these are the only two judgements allowed (alternatives are not allowed to be judged as equivalent), we have $I_{12} + I_{21} = I$. If $I_{12} = I$ and $I_{21} = 0$ or vice versa, then we have unanimous judgements; any other result we refer to as controversial. A unanimous result has a degree of controversy of 0 (all decision-makers make the same judgement), and the maximum possible degree of controversy holds for $I_{12} = I_{21} = I/2$ (equal numbers of decision-makers prefer A_{j1} over A_{j2} and *vice versa*.)

The central assumption of our model is that controversy can be interpreted as uncertainty, since we assume that a controversial result results from a lack of sufficient information, and that individual judgements are then essentially random. Similarly, we deem unanimity to correspond to a maximally certain result. In this case, we assume that all decision-makers have enough information (or prior knowledge) to determine the correct judgement. This assumption is intuitive, since the larger the majority of decision-makers that favours one alternative over another, the more certain this result one would assume this result to be.

With this assumption, we can measure the degree of uncertainty (controversy) inherent in each pairwise comparison simply by counting judgements. Comparisons with a high degree of uncertainty are assumed to be lacking in information. In practice, the two alternatives which receive a controversial comparison must then be enhanced with additional information that is relevant to the comparison. Conversely, those judgements which have a low uncertainty can be accepted as they are. After the additional information has been provided, the controversial (i.e. uncertain) comparisons can be repeated; if the additional information is sufficient, then we can expect to obtain a unanimous (i.e. certain) result.

This approach requires the specification of one parameter, namely the threshold that distinguishes controversial comparisons from uncontroversial ones. In an application with 10 decision-makers, for example, the choice might be made to treat judgements that are 5-5 or 6-4 as controversial, and all others as uncontroversial.

5. EXPERIMENTAL STUDY

In order to gain some experience with the model, we performed a small experiment. We considered a hypothetical secretary with several years experience who is considering starting her own business. We generated seven business ideas and gave them to a group of 18 non-experts for evaluation. The evaluation criterion "*Level of investment needed*" was used. Only one evaluation criterion was used, since additional criteria would have increased the workload for the test subjects without providing any further insights into the model. Of course, in a real-life situation, many more important criteria would have to be considered. The following business ideas were used for the experiment:

A. Renting out a conference room to small businesses
B. Writing business presentations
C. Healthy workplace service
D. Monthly rental of works of art to hotels, restaurants and cafes
E. Training for improved customer service
F. Writing book summaries
G. Dry cleaning delivery service

The experiment consisted of the following steps:

1. The decision-makers use pairwise comparisons of alternatives with respect to the evaluation criterion to generate individual rankings.
2. The comparisons are aggregated to form a Markov chain. The Markov chain is solved to obtain an overall ranking.
3. The controversial comparisons are identified.
4. The decision-makers are given additional information on the alternatives which are involved in controversial comparisons.
5. The decision-makers repeat the comparisons which were controversial using the new information.
6. A new Markov chain is constructed and solved to obtain a revised ranking.

For simplicity, each decision-maker was assigned the same coefficient. Again, in practice, different decision-makers may be assigned different coefficients to reflect their varying levels of experience or expertise. The choice of coefficient has no effect on our results.

The solution of the Markov chain was computed and is shown in Table 1. In this example, alternative *F "Writing book summaries"* has the highest probability value and was therefore ranked first, and alternative *A "Renting out a conference room to small businesses"* received the lowest probability value and was therefore ranked last.

Table 1: Computed ranking for the seven alternatives

Value	Idea #
0.583	*F*
0.215	*B*
0.068	*E*
0.054	*C*
0.043	*G*
0.024	*D*
0.013	*A*

Table 2 shows the distributions of the judgements obtained. The alternatives in the rows were preferred over those in the columns. Thus, for example, five decision-makers preferred alternative *D* over alternative *C*, and 13 preferred alternative *C* over alternative *D*. We considered a comparison to be controversial if it came out as 9-9 or 10-8. Using this value, only one of the 42 comparisons – namely between alternatives *E* and *C* – was controversial.

Table 2: Distribution of individual judgements

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	-	1	4	5	4	0	4
B	17	-	15	16	15	4	16
C	14	3	-	13	9	1	13
D	13	2	5	-	7	0	7
E	14	3	9	11	-	3	11
F	18	14	17	18	15	-	16
G	14	2	5	11	7	2	-

In step 4 of the experiment, the decision-makers were given information on the investments that would need to be made for alternatives *E* and *C*. This resulted in a comparison which was 16-2 in favour of *E*.

After repeating the Markov chain computation, a new ranking was obtained, as shown in Table 3. Alternatives *G* and *C* have exchanged positions, otherwise, the ranking remains unchanged. It is worth noting that, of the seven alternatives, only two had to be developed further in order to obtain a result which satisfied the certainty criterion.

Table 3: Final ranking

Value	Idea #
0.582	*F*
0.215	*B*
0.081	*E*
0.044	*G*
0.041	*C*
0.024	*D*
0.013	*A*

It is difficult to make general predictions about the development savings that our approach can make possible. In the extreme case that all alternatives are contained in at least one uncertain comparison, then no savings are possible, because all alternatives must be developed. On the other hand, if all comparisons are unanimous or nearly unanimous, the model tells us that our selection has a high degree of certainty, and this is achieved without any development cost.

There are a few situations in which the unanimity of a comparison can be predicted. An alternative which is obviously superior to the others will yield unanimous comparisons. The same is true for an alternative which is obviously inferior to all the others. On the other hand, an evaluation criterion which is ambiguous will tend to yield uncertain comparisons, since the decision-makers may base their judgements on different interpretations of this criterion.

6. CONCLUSIONS

This paper presents a new model for localising uncertainty in multi-person multi-criteria decision problems. The model is designed for use with a ranking algorithm that utilises pairwise comparisons of alternatives. It is based on the assumption that controversial comparisons are a symptom of uncertainty.

The model is useful in cases with localised uncertainty, i.e. in which decision-makers disagree on a pairwise comparison for a limited number of alternatives. In such cases, the model suggests which alternatives need to be developed in order to reduce the uncertainty in the ranking. In the extreme case that many or all alternatives are part of uncertain comparisons, then no development savings are possible. In the other extreme case, in which all comparisons are unanimous or nearly unanimous, the model determines that no development is necessary.

The model is still at an early stage of development, and experience with different MPMCDPs needs to be gathered. In addition, we would like to develop an overall uncertainty score for a ranking which aggregates the uncertainties of the individual comparisons in an appropriate manner.

Our approach assumes that uncertainty is caused by a lack of information. However, experience shows that ambiguously formulated evaluation criteria can have the same effect. We therefore plan to develop a method for detecting these ambiguities in order to prompt a re-formulation.

Uncertainty is a significant problem in practice, since the cost of making an incorrect selection in the early stages of the innovation process can be considerable. We believe that our approach can make a contribution to the efficiency and the reliability of innovation processes.

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