PROXEL-BASED SIMULATION OF A WARRANTY MODEL

Sanja Lazarova-Molnar and Graham Horton
Institut für Simulation und Graphik, Fakultät für Informatik
Otto-von-Guericke-Universität, Universitätsplatz 2
D-39106 Magdeburg, Germany
E-mail: sanja@isg.cs.uni-magdeburg.de

KEYWORDS

Proxel, supplementary variables, stochastic Petri net, state space-based simulation.

ABSTRACT

The Proxel method is a state space-based approach to the simulation of discrete stochastic models. It implements the method of supplementary variables in an algorithmic, rather than mathematical way, without using partial differential equations and is able to solve a wide class of stochastic models. The approach has proven to be very competitive with discrete-event simulation for a class of models that is used in reliability modelling.

In this paper we report on our experience with the application of the proxel-based method to a problem provided by the DaimlerChrysler Corporation. The model that portrays the problem describes the correlation between system reliability and warranty costs for automobiles. Until recently, this model was analysed using discrete-event simulation, which resulted in very time-consuming computations. By contrast, the proxel-based method achieved comparable accuracy within a few seconds of computation time.

INTRODUCTION

The analysis of stochastic models is a common task in reliability modelling. The problem of long and expensive simulations is always present and thus also the need for faster and cheaper approaches. Models which are stiff, or whose measures have a large variance, require a large number of replications when Monte Carlo simulations are used; the model considered in this paper required 20 to 30 hours of computation time.

Proxel-based methods are a new way of analysing stochastic models, introduced in (Horton 2002). They are a state space approach, using supplementary variables to model the non-Markovian activities. They thus suffer from the state space explosion problem. However, since they are effectively a discretisation of a system of differential equations, they provide a much more controlled convergence towards the solution as the computation progresses. This fact was exploited in this paper to achieve results of comparable accuracy in just a few seconds.

In this paper, proxel simulation is very briefly described, followed by the warranty model that was studied. Results of an experiment are given, which illustrate the behaviour of the method and the application of a corresponding software tool to the problem.

PROXEL-BASED SIMULATION

The proxel-based method (Horton 2002; Lazarova-Molnar and Horton 2003, Lazarova-Molnar and Horton 2004) is a state space-based approach which uses supplementary variables to model non-Markovian activities. It is a numerical algorithm which proceeds in discrete time steps of size *dt*, as opposed to continuous-time approaches such as are described in (German 2000). One important definition in the proxel-based simulation is the one of the term "*state"* which is the vector composed of the *discrete state of the system* and the *age intensities* of the state changes that are possible in those states. The age intensity of a state change is the time that the model has been waiting in a discrete state for that state change to occur. The probability of the state change happening within a time step of length *dt* can be computed using the *instantaneous rate function* (*IRF*) given the age intensity as a parameter.

The proxel, as a basic unit of computation, describes a state of the model in a complete and minimal way. A proxel P is defined as the following:

$$P = (S, t, R, p)$$

$$S = (\textit{Discrete State of the System, Age Intensity Vector})$$

$$R = (\textit{State1, State2,…, StateN})$$

where t is the simulation time, and p denotes the probability that the model is in state S at time t, given that it has been reached from the initial state through the sequence of states R. The *Age Intensity Vector* is a vector containing the age intensities of the enabled state changes in state S. These are needed for a complete definition of the state of the model, because the probabilities of the next state changes are dependent on how long the concurrent activities have been active.

A proxel simulator stores a dynamic list of such proxels and proceeds by generating the set of possible successor proxels at a new time step and computing the probability of these new proxels using the IRF of the corresponding activities. Essentially, the method is equivalent to a discrete-time Markov chain approach in which states are generated on-the-fly. Detailed descriptions of the method can be found in (Lazarova-Molnar and Horton 2003, Lazarova-Molnar and Horton 2004).

DESCRIPTION OF THE WARRANTY MODEL

The model that was analysed was used to predict the warranty costs for automobiles using different warranty strategies. These strategies contained a race condition between a time-based and a mileage-based expiration threshold, whereby the simulation time unit t is measured in miles, and physical time is converted into an equivalent number of miles. Figure 1 shows a stochastic Petri net of this model.

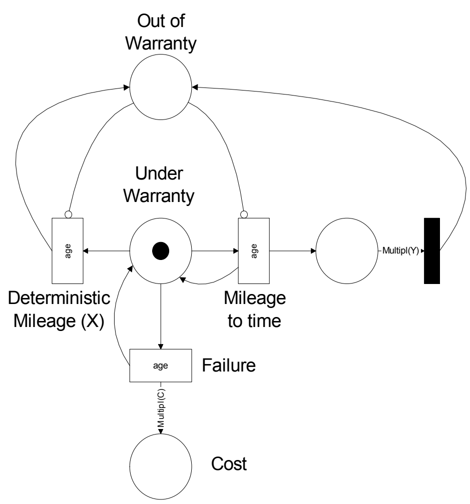

Figure 1: Petri net of the warranty analysis model

The goal of the analysis of this model was to predict the manufacturer's costs for different types of vehicles and analyse different warranty strategies. As shown in Figure 1, the warranty runs out whenever one of the two conditions was satisfied: either the warranty period has run out or the warranty mileage was reached. During the warranty period, the manufacturer incurs costs whenever the vehicle fails and must be repaired. These costs are a decision factor in constructing the warranty strategy.

The discrete-event simulation of this model resulted in long computation times, owing to the rarity of the failures. The proxel-based method, as previously mentioned, is less sensitive to the stiffness of models and results in much shorter computation times.

The model was analysed using the following parameters:

- Y – the number of years under warranty,
- X – the mileage under warranty, and
- C – average costs per failure

and the following distribution functions:

- f – failure distribution function, and
- g – time to mileage distribution function.

In the proxel-based simulation, the number of years was included in the state vector in the same manner as the age intensities are; therefore the model results in only two discrete states, *"under warranty"*(U) and *"out of warranty"*(O). The state vector in the general case has the following form:

(*discrete state, age intensity of the failure transition, number of years passed*)

with the initial state being:

$(U, 0, 0)$.

The things that could happen in the next time step are:

(1) one year could have passed, which probability can be computed from the instantaneous rate function of g, or

(2) failure could have happened, which probability can be computed from the instantaneous rate function of f (the costs are then incremented), or

(3) global simulation time (mileage) could have reached X, which means end of simulation, or

(4) nothing, where the probability is one minus the probabilities of everything else happening, whereby the age intensities are incremented appropriately.

The corresponding states are the following:

(1) $(O, dt, 1)$,

(2) $(U, 0, 0)$,

(3) $(O, dt, 0)$, and

(4) $(U, dt, 0)$.

The proxel-based simulation traces the flow of probability and the accumulation of costs. The goal is to provide an approximation for the expected costs. The approximations differ in their accuracy based on the size of the time step being used. The solution values, however, converge monotonically towards the true solution as the size of the time step decreases. This makes it possible to perform an extrapolation of solution values obtained with larger time steps and thus obtain a better approximation. The fact that the simulation can be run using larger time steps means an enormous saving of computation time.

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EXPERIMENTS

In the following we will provide an example computation for the warranty analysis model. For this purpose we will use the following (fictitious) values for the required parameters:

- C = 1000 $
- *f* ~ *Exponential*(0.00001 per mile)
- *g* ~ *Weibull*(4500 miles, 1.5)

The corresponding values for X and Y are given in the following table:

X (miles)	5000	10000	15000	20000
Y (years)	1	2	3	4

The user interface of the special-purpose proxel-based tool for the first simulation (for the first values of X and Y) is shown in Figure 2.

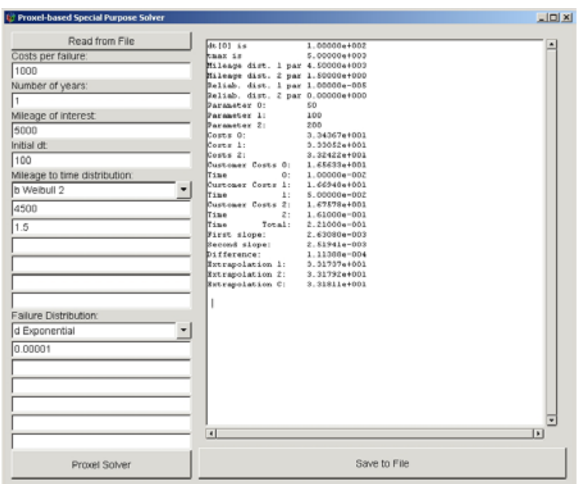

Figure 2: GUI of the proxel-based special-purpose solver

Using an initial value of 100 for *dt*, the approximations for the costs are given in Figure 3.

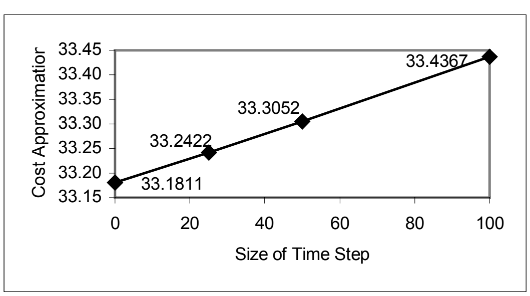

Figure 3: Costs approximations using different time steps and extrapolation

In Figure 3, the values for *dt* = 100, 50, 25 are obtained using the proxel-based simulation, whereas the value for *dt* = 0 is obtained using Lagrange's formula for extrapolation. The overall computation time for this simulation was 0.24 seconds.

In Figure 4 the results of the same simulation are shown, using an initial time step of 300. The computation time was 1.02 seconds. This illustrates how the initial size of the time step influences the solution.

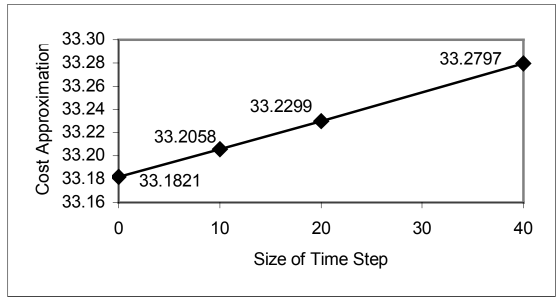

Figure 4: Solution values for the costs with respect of the size of the time step

X (miles)	5000	10000	15000	20000
Y (years)	1	2	3	4
Costs ($)	33.1675	72.5313	112.7430	153.3080
Comp. Time (s)	0.020	1.462	11.456	46.967

Figure 5: Estimate for the cost using initial *dt* = 500 miles

X (miles)	5000	10000	15000	20000
Y (years)	1	2	3	4
Costs ($)	33.1442	72.4817	112.6660	153.2040
Comp. Time (s)	0.010	0.160	1.212	4.757

Figure 6: Estimate for the cost using initial *dt* = 1000 miles

From Figures 5 and 6, it can be seen that the saving in computation time when using larger time step is big and the accuracy affected only a little. This also proves that the proxel-based method is very flexible and especially suitable when a rough approximation is needed quickly. We are also working on a better way of measuring and controlling the accuracy of the proxel-based simulation.

From the cost analysis of the model, it could be discovered how the costs vary for the four different cases (values of X and Y). In the example presented, the change of the costs is linear, as shown in Figure 7, which was to be expected, since the failure distribution is exponential, i.e. it has a constant instantaneous rate function.

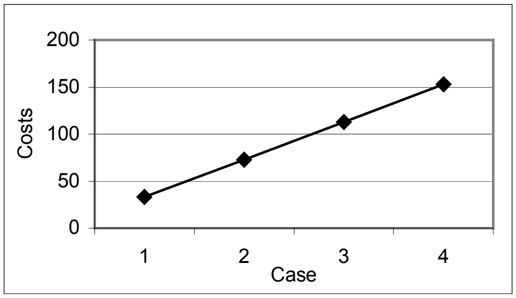

Figure 7: Development of costs for the four different cases from Figure 6

SUMMARY AND OUTLOOK

This positive experience with the proxel-based method opened the doors to our further cooperation with DaimlerChrysler with respect to analysing their models with the proxel-based method. The short computation times are significant from the point of view of providing finer analysis of the costs within reasonable amounts of time, which was almost impossible using the discrete-event simulation approach. The latter approach was only able to do rough prediction of the costs for predetermined coarse points, where each simulation was very expensive, with duration of about 20-30 hours. In a time significantly less than that, the proxel-based simulation could provide a more accurate smooth prediction and therefore provide a better decision concerning the warranty strategy.

This is, however, only one way in which the proxel-based simulation is superior to the discrete-event simulation approach. Another one would be getting rough approximations of the costs within even smaller amounts of time. The flexibility of the method makes it possible to compromise the accuracy with the computation time when only a vague idea of the solutions is needed. The sacrifice (in terms of accuracy), however, is not too big as it was shown in Section 4.

The accuracy of the results could also be improved by using more points for the extrapolation of the solution values. One of the next steps in our research is to provide easily controllable accuracy in the size of the time step and the number of interpolation points.

The success of the approach has also led us to construct a proxel-based simulation tool for analysing the times to failure for non-trivial basic events in fault trees (Lazarova-Molnar and Horton, 2003a). These models are small, but a high degree of accuracy is required; these are ideal conditions for the proxel method.

In a more general context, models are larger, and the explosion of the state space becomes a dominant factor. For such cases, we are working on heuristics such as adaptive time steps to reduce the number of variables to be generated and stored.

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AUTHOR BIOGRAPHIES

SANJA LAZAROVA-MOLNAR was born in Skopje, Macedonia and went to “Sts. Cyril and Methodius” University in Skopje where she obtained her degree in Computer Science. She continued her studies in Magdeburg from where she obtained her Masters degree in Computational Visualistics. At the moment she is a member of the Simulation and Modelling group at the university in Magdeburg working towards her PhD in the field of Simulation. She can be reached at sanja@isg.cs.uni-magdeburg.de

GRAHAM HORTON studied Computer Science at Erlangen University, Germany and graduated with a Masters degree in 1989, followed by PhD and Habilitation degrees in 1991 and 1998, respectively. Since 2001 he is Professor of Simulation at the Computer Science Department at the University of Magdeburg, Germany. He can be reached at graham@cs.uni-magdeburg.de